Skin in the Game: Risk Analysis of Central Counterparties

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Background: Central Clearing

- Central clearing of standardized OTC derivatives has been one of the main components of the G20 reform program after the global financial crisis (GFC) of 2007-2009 (Ghamami and Glasserman, 2017).[2]

- The Covid-19 crisis revealed that the secondary market for U.S. Treasuries can become dysfunctional in part due to the constraints on the capacity of dealers that intermediate this market (Duffie, 2020). Broadening central clearing mandates in government securities markets has been one of the main elements of the 2021 G30 reform program.

- Central clearing has the potential to reduce the interconnectedness of the financial system and improve transparency. It can also help mitigate counterparty credit risk through multilateral netting (Duffie and Zhu (2011); Cont and Kokholm (2014)). Central clearing may also reduce the pressure on intermediaries' balance sheets (Baranova et al., 2023).

[2]References in this presentation have been *hyperlinked*.

Background: Central Counterparties

- CCPs require effective governance, regulatory oversight, and highly robust risk management frameworks. Otherwise, increased use of CCPs may create financial stability risks (Bernanke (2011); Tucker (2014)). The failure of a systemically important CCP can be disastrous.
- The right design and regulation of CCPs continue to generate debate among industry participants, government officials, and the public.
- In 2019 and 2020, major buy-side and sell-side firms called for regulatory action to make clearinghouses safer. The *2019-2020 industry paper* consisted of a number of recommendations. One of the main recommendations was: "requiring CCPs to make material contributions of their own capital to the default waterfall in two separate tranches."

Multilayered Default Waterfall

- The first layer of protection against default losses is provided by the initial margin (IM) posted by the defaulting member.
- If default losses exceed the defaulting member's IM, its prefunded default fund (DF) assets are used to cover any additional losses. If losses exceed the defaulter's IM plus DF, the CCP makes a contribution to offset the remaining losses. This CCP capital contribution is often referred to as skin-in-the-game (SITG), denoted by S.
- DF assets of surviving members are used against potential remaining losses. These losses can be mutualized and allocated across members proportional to their DF contributions.
- Once the total DF is exhausted, the CCP may use different recovery mechanisms to restore its funding resources. These include: (i) an additional capital contribution by the CCP, denoted by $\tilde{S}$; (ii) additional capped DF contributions by surviving members; and (iii) other recovery measures, such as variation margin haircuts.

Initial margin of defaulting member

Default fund contribution of defaulting member

CCP skin-in-the-game : CCP capital contribution

Default fund contributions of (surviving) members

CCP skin-in-the-game : CCP capital contribution

Default fund replenishment by (surviving) members

Other recovery measures: variation margin haircuts, …

Resolution regime

Monolayer Default Waterfall

- We also analyze the *monolayer* default waterfall where the IM pool is used for loss mutualization as a separate layer of DF does not exist in addition to IM in some CCPs.
- This case is important as some of the systemically important securities CCPs in the U.S. operate under this structure. The broader central clearing proposal in the U.S. may take place under the monolayer default waterfall at the Fixed Income Clearing Corporation (FICC).
- Large derivatives CCPs do not mutualize the pool of IM to cover defaulting member losses. It is the default fund, the layer of collateral collected in addition to IM, that can be mutualized to cover losses.

Goal of the Paper

- As shown in recent surveys (Thiruchelvam (2022); Walker (2023)): (i) SITG is often a very small fraction of member prefunded resources. For instance, SITG is 1 percent of the default fund at LCH for interest rate swaps; (ii) SITG levels vary widely across CCPs; and (iii) policymakers do not have a quantitative methodology for specifying SITG and evaluating its sufficiency.
- The goal of our research is to address these shortcomings.
- We introduce a robust SITG design framework. Unlike bank regulation, CCP regulation is mostly principles-based (Ghamami, 2015). Capital regulation of CCPs may not adequately correspond to their risk profiles. Our proposed formulations of SITG can be viewed as risk-based lower bounds on minimum CCP capital requirements.

Risk Management Agency Problems

- CCPs can be viewed as counterparty credit risk insurance providers. The classical moral hazard problem here is that members may be incentivized to take more counterparty credit risk. A well-designed loss mutualization scheme and adequate collateral requirements could mitigate this moral hazard problem.
- Given that default losses can be mutualized among surviving members, in the absence of adequate levels of SITG, CCPs may not be incentivized to properly monitor counterparty credit risk. CCP risk management practices could subsequently become questionable. A well-designed SITG can mitigate this variation of moral hazard.
- This agency problem can become subtle at member-owned CCPs. Drawing on the work of Hart and Moore (1996) and Hansmann (2013), we show that managers at a members' cooperative may not be adequately incentivized to put in place robust risk management frameworks. Member-owned CCPs could face collective decision-making challenges that may lead to insufficient levels of SITG. This problem can be exacerbated under CCPs with heterogeneous membership.

Sketch of the Proposal & Summary of Our Findings

- Conditional on a member's default, when $S = 0$, surviving member DF assets are more exposed to losses compared to losses that the CCP could face under member prefunded resources. We introduce *incentive compatibility constraints* (ICCs) and formulate SITG to mitigate risk management moral hazard problems.

- S can be formulated as a percentage of total DF, denoted by D. Consider the CCP's tail exposure to each member conditional on its default. Suppose that member 1 creates the largest tail exposure. The ratio of the CCP's largest tail exposure to aggregate tail exposures is called *concentration ratio*, c_1. When

$$S = (1 - c_1)D,$$

some of the overarching ICCs are satisfied, and CCP-member risk management incentives become more aligned.

Sketch of the Proposal & Summary of Our Findings

- When $\tilde{S} = 0$, member unfunded DF contributions are more exposed to losses compared to CCP loss exposures. This can also distort risk management incentives. The moral hazard problem can be mitigated by formulating $\tilde{S}$ that satisfies a set of ICCs. Similar to S, $\tilde{S}$ can also be formulated as a percentage of D.

- We use the *threshold exceedances* approach (McNeil et al., 2015) from extreme value theory (EVT) to model the conditional distribution of losses in excess of IM by the Pareto distribution. This produces our SITG design framework in its most general form.

- Total SITG can also be expressed as a percentage of D. *Our numerical studies indicate that for realistic parameters, this leads to SITG levels above 15-20 percent of D.* This in turn leads to estimates of lower bound for CCP equity capital in terms of D.

Summary of Our Findings

- Monolayer CCPs may need to hold significantly higher levels of SITG to mitigate risk management agency problems. We can approximate the ratio of the monolayer CCP SITG to the multilayered CCP SITG under similar ICCs. This ratio is roughly equal to the ratio of total IM to total DF under the multilayered default waterfall. In practice, IM can be 10 times or more larger than DF.

- FICC's capital contribution is less than 1 percent of its members' prefunded resources. Our results indicate that higher levels of SITG may be required to mitigate potential risk management incentive distortions.[3]

- Our findings have also implications for the adequacy of bank capital requirements for exposure to CCPs. Our numerical studies show that *CCP risk capital rules* can be improved as central clearing risks may be underestimated in the current regulatory regime.

[3] Woodall (2021) has documented that: "Its $61 million contribution represented 0.14% of total prefunded resources. With 208 members as of end-September, the average individual clearing member contribution was $208 million."

Tail Exposures

- Consider a CCP that clears transactions in an asset class for N members indexed by $i = 1, ..., N$. U_i represents the exposure of the CCP to member i over a given *risk horizon*. U_i captures in part member i's portfolio value changes over the risk horizon.

- Member i contributes an initial margin M_i to the CCP. M_i is a a quantile of U_i for confidence level $1 - q$; $q \leq 0.01$. The CCP's exposure to member i net IM is $(U_i - M_i)^+ = \max(U_i - M_i, 0)$. The magnitude of this exposure in *extreme but plausible scenarios* is modeled using a risk measure ρ associated with $(U_i - M_i)^+$ at confidence level $1 - q_D$,

$$E_i = \rho_{q_D}\Big((U_i - M_i)^+\Big),$$

  where $q_D < q$. D_i represents the contribution of member i to DF, and D denotes $D = \sum_{i=1}^{N} D_i$, the size of the total DF.

Default Fund

- Regulatory guidelines require that DF covers potential losses incurred due to a given number of member defaults, at least one and often two for systemically important derivatives CCPs. Denoting by $E^{(i)}$ the ith the largest exposure, *cover-one* DF leads to a prefunded default fund given by,

$$D = \max\left(E_i, \quad i = 1..N\right) = E^{(1)}.$$

  Cover-two DF is intended to cover the simultaneous default of two members that would jointly create the CCP's largest (tail) exposure. Cover-two DF can be formulated as $D = E^{(1)} + E^{(2)}$.

- Some derivatives CCPs allocate DF to members proportional to tail exposures,

$$D_i = D \quad \frac{E_i}{\sum_{j=1}^{N} E_j}.$$

Capital Contribution to the Default Waterfall

In the absence of SITG regulation, investor-owned CCPs may not be incentivized to make capital contributions to the default waterfall. Conditional on the default of member j, the loss to the CCP can be written as

$$L = \min\left\{(U_j - M_j - D_j)^+, S\right\} + \min\left\{(U_j - M_j - D - S)^+, \tilde{S}\right\}. \quad (1)$$

V and ϕ denote the CCP's average clearing volume over a given period of time and the clearing fee. The CCP's expected net profit can then be approximated by

$$\phi V - E[L]. \quad (2)$$

The CCP maximizes expected net profits by choosing optimal levels of S and $\tilde{S}$. In the absence of capital constraints, the CCP solves this problem by setting $S = \tilde{S} = 0$.

Skin in the Game: First Layer

- If member j defaults, the potential loss to the DF assets of a non-defaulting member can be written as

$$L_i^j = D_i \quad \min\left(\frac{(U_j - M_j - D_j - S)^+}{D - D_j},\ 1\right). \tag{3}$$

  When $S = 0$, the probability that non-defaulting members take a loss is larger than q_D,

$$P(U_j > M_j + D\frac{E_j}{\sum_{k=1}^N E_k}) \geq P(U_j > M_j + E_j) = q_D.$$

  In short, for any $i \neq j$, setting $S = 0$, gives $P(L_i^j > 0) \geq q_D$.

- Conditional on the default of j, when $S = 0$, the potential loss to the CCP in the presence of IM and DF is $L_0^j = (U_j - M_j - D)^+$.

Skin in the Game: First Layer

Since $P(L_0^j > 0) \leq q_D$, when $S = 0$, members are more likely than the CCP to incur losses

$$P(L_i^j > 0) \geq q_D \geq P(L_0^j > 0).$$

This moral hazard problem can be mitigated by formulating S that satisfies the following ICC

$$P(L_i^j > 0) \leq q_D. \tag{4}$$

Given that

$$P(L_i^1 > 0) = P(U_1 - M_1 > D_1 + S), \text{ and } P(L_0^1 > 0) = P(U_1 - M_1 > D),$$

setting $S = D - D_1$ gives

$$P(L_i^1 > 0) = P(L_0^1 > 0) = q_D. \tag{5}$$

$S = (1 - c_1)D$ aligns *large* counterparty default loss probabilities. Also, under the EVT-based framework, q_D becomes an upper bound on member loss probabilities, i.e., ICC (4) will hold.

Skin in the Game: Second Layer

- $\tilde{S}$ can be viewed as a buffer against losses to unfunded DF contributions. Conditional on j's default, total loss to i's prefunded and unfunded DF becomes

$$\tilde{L}_i^j = L_i^j + \left(U_j - M_j - S - D - \tilde{S}\right)^+ \frac{D_i}{D - D_j}. \tag{7}$$

  When unfunded default funds are capped by a multiple of D_i, βD_i; $\beta > 0$, the second term on the right side above is replaced with the minimum of it and βD_i. We have $P(\tilde{L}_i^j > D_i) = P(U_j - M_j > D + S + \tilde{S})$.

- Consider a *target loss probability* $\tilde{\pi}$; $\tilde{\pi} < q_D$. $\tilde{S}$ is formulated to satisfy the following ICC

$$P(\tilde{L}_i^j > D_i) \leq \tilde{\pi}. \tag{8}$$

  S and $\tilde{S}$ satisfy the overarching ICCs (4) and (8).

Skin in the Game: Second Layer

- Conditional on j's default, when $\tilde{S} = 0$, the CCP's potential loss in excess of S and prefunded and unfounded resources is
$$\tilde{L}_0^j = \left(U_j - M_j - D - S - \beta(D - D_j)\right)^+.$$
So, when $\tilde{S} = 0$, we have $P(\tilde{L}_i^j > D_i) > P(\tilde{L}_0^j > 0)$. $\tilde{\pi}$ will be chosen such that ICC (8) mitigates this moral hazard problem.
- Suppose that $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$. Setting $\tilde{S} = \beta(1 - c_1)D$ results in
$$P(\tilde{L}_i^1 > D_i) = P(\tilde{L}_0^1 > 0) = P\big(U_1 - M_1 > D + S + \beta(D - D_1)\big), \qquad (9)$$
i.e., it fully aligns CCP-member largest counterparty default risk management incentives. Under our EVT-based framework,
$$P(\tilde{L}_i^j > D_i) \leq P(\tilde{L}_i^1 > D_i),$$
for $j \neq i, 1$. So, (9) along with the above inequality gives ICC (8).

Modeling Tail Risk

- We model the conditional distribution of losses in excess of IM as a Pareto distribution whose tail exponent (shape parameter) $\alpha > 1$ quantifies the heaviness of the tail.
- The CCP's exposure to member i conditional on its default satisfy

$$P(U_i - M_i > x |\ U_i \geq M_i) = \left(\frac{\kappa_i + x}{\kappa_i}\right)^{-\alpha} = 1 - F(x;\kappa_i,\alpha),$$

$$\text{where} \qquad M_i = VaR_q(U_i), \quad \text{and} \qquad F(x;\kappa,\alpha) = 1 - \left(\frac{\kappa + x}{\kappa}\right)^{-\alpha}$$

  is the Pareto distribution with the tail exponent $\alpha > 1$ and scale parameter $\kappa > 0$.
- We assume that default loss distributions can be represented with some tail exponent α and a scale parameter κ_i that may vary across members. This gives

$$\frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{\kappa_i}{\sum_{j=1}^{N} \kappa_j}.$$

  This ratio is denoted by c_i, (c_1 is the concentration ratio).

Pareto-based SITG: First Layer

- We show that

$$P(L_i^j > x) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(c_1 + \frac{S}{E_j} + \frac{x}{E_j}\frac{(1-c_j)}{c_i}\right)\right]^{-\alpha}.$$

  So, the highest level of tail risk corresponds to the default of member 1, $P(L_i^j > x) \leq P(L_i^1 > x)$, for any $j \neq i, 1$.

- We can compute S which corresponds to a given target loss probability π. Suppose that $\pi = P(L_i^1 > 0)$. Solving for S yields,

$$P(L_i^1 > 0) = \pi \iff S = \left(\frac{(\frac{q}{\pi})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} - c_1\right)D. \quad (10)$$

  Our objective is to lower loss probabilities to achieve ICC (4). Setting $\pi \leq q_D$ satisfies this criterion. Aligning *largest* counterparty default loss probabilities is achieved by choosing S such that $\pi = P(L_0^1 > 0) = q_D$. Doing so gives $S = (1 - c_1)D$.

Pareto-based SITG: Second Layer

- The likelihood that the loss to the CCP exceeds S and members prefunded and unfunded DF takes its maximum conditional on the default of member 1. Similarly, the probability that the loss to a nondefaulting member exceeds its prefunded DF assets is largest conditional on the default of member 1.

- Given our formulation of S with $\pi \leq q_D$, we can specify $\tilde{S}$ that corresponds to a target loss probability $\tilde{\pi}$. Suppose that $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$. Solving for $\tilde{S}$ gives

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{\left(\frac{q}{\tilde{\pi}}\right)^{1/\alpha} - \left(\frac{q}{\pi}\right)^{1/\alpha}}{\left(\frac{q}{q_D}\right)^{1/\alpha} - 1} + c_1 - 1 \right) D, \tag{11}$$

  where $\tilde{\pi} < \pi \leq q_D$. When $\tilde{\pi}$ is set as $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$, the above formulation results in $\tilde{S} = \beta(1 - c_1)D$.

- Designing S and $\tilde{S}$ with π and $\tilde{\pi}$ ensures that ICC (8) is satisfied, $P(\tilde{L}_i^j > D_i) \leq P(\tilde{L}_i^1 > D_i) < q_D$, for any $j \neq i, 1$.

Capital Regulation

- Our framework gives the following lower bound for minimum capital requirements,

$$S + \tilde{S} = \left[\frac{\left(\frac{q}{\tilde{\pi}}\right)^{1/\alpha} - 1}{\left(\frac{q}{q_D}\right)^{1/\alpha} - 1} - 1 \right] D.$$

  Recall that $\pi = P(L_i^1 > 0)$ and $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$ satisfy $\tilde{\pi} < \pi \leq q_D < q$.

- Total SITG below 15-20 percent of DF cannot be produced in the realistic part of the parameter space. This contrasts with current practice.

- Huang (2019) estimates an average SITG of about USD 38 million and an average IM of about USD 14 billion across 9-10 CCPs. When DF is 10 percent of IM, SITG becomes 2.7 percent of DF. This level of SITG may not adequately mitigate risk management incentive distortions.

Monolayer Default Waterfall

- In the absence of SITG, the exposure of a *monolayer CCP* conditional on j's default can be written as

$$\check{L}_0^j = (U_j - M)^+,$$

where $M = \sum_{i=1}^N M_i$. To compare monolayer and multilayered waterfalls, we (naturally) assume that U_j's are drawn from the same distribution under both waterfall structures. Recall that $L_0^j = (U_j - M_j - D)^+$. It is often the case that $P(\check{L}_0^j > x) \leq P(L_0^j > x)$ for $x \geq 0$.

- Conditional on j's default, potential losses to the IM of i can be written as

$$\check{L}_i^j = M_i \min\left(\frac{(U_j - M_j - \check{S})^+}{M - M_j}, 1\right),$$

in the scenario where member losses cannot exceed M_i.

Comparing Monolayer and Multilayered Default Waterfalls

- Under the Pareto assumption, we have

$$P(\check{L}_i^j > 0) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\frac{\check{S}}{E_j}\right]^{-\alpha}.$$

  This implies that $P(\check{L}_i^j > 0) \leq P(\check{L}_i^1 > 0)$ for any $j \neq i, 1$.

- When $\check{S} = 0$, we have $P(\check{L}_i^j > 0) = P(U_j > M_j) = q$. Since $P(\check{L}_0^j > 0) = P(U_j > M_j + \sum_{i \neq j} M_i)$, members can incur disproportionately larger losses. To align largest counterparty default loss probabilities, the CCP should contribute $\check{S} = M - M_1$ to the default waterfall,

$$P(\check{L}_i^1 > 0) = P(\check{L}_0^1 > 0). \tag{12}$$

  Condition (12) is analogous to ICC (5) that is satisfied under $S = (1 - c_1)D$ in the multilayered CCP.

Higher SITG levels or restructuring the monolayer default waterfall (?)

$\check{S} = M - M_1$ can be written as

$$\check{S} = (1 - c_1)M \tag{13}$$

when $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student- t distribution with $\nu > 1$ degrees of freedom. Under (13), the following ICC is satisfied,

$$P(\check{L}_i^j > 0) \leq P(\check{L}_0^1 > 0), \tag{14}$$

for any $j \neq i, 1$. We just provided *incentive compatibility arguments* initially used in the design of SITG that led to $S = (1 - c_1)D$. In short, incentive compatibility constraints require

$$\check{S} = \frac{M}{D} S.$$

Capital contribution of the monolayer CCP may need to be several multiples of that of a similar CCP with multilayered default waterfall.

Member-Owned CCPs

Suppose that $\psi_i V_i$ represents member i's gross profit from its trades in a volume of V_i that have been cleared through the CCP. Set $V = V_1 + V_2 + ... + V_N$. Suppose that all members receive an equal share of the CCP's profit. Then, conditional on the default of j, member i's expected net profit can be written as

$$\frac{\phi V - E[L]}{N-1} + \left(\psi_i V_i - E[\tilde{L}_i^j]\right),$$

the term inside the parentheses can be viewed as member i's *consumer surplus*.[4] $\tilde{L}_i^j$ is the total loss to member i's prefunded and unfunded DF defined in (7). Member i maximizes expected net profit by choosing optimal levels of S and $\tilde{S}$,

$$\frac{\phi V}{N-1} + \psi_i V_i - \left(\frac{E[L]}{N-1} + E[\tilde{L}_i^j]\right). \tag{15}$$

[4] In any market, total surplus can be viewed as the sum of total producer surplus and total consumer surplus (Hart and Moore, 1996).

Regulating SITG at Member-Owned CCPs

- Consider expected losses in (15). CCP managers' expected loss $E[L]$ can be viewed as an increasing function of S and $\tilde{S}$ while member i's expected loss $E[\tilde{L}_i^j]$ can be viewed as a decreasing function of S and $\tilde{S}$. So, an optimal first and second layer SITG could be positive.

- Member and CCP expected net profit functions highlight the adverse impact of membership heterogeneity on SITG levels. $E[\tilde{L}_i^j]$ can be viewed as increasing functions of D_i. Since *larger members* contribute more to the DF, their optimal levels of SITG can be larger than that of *smaller members*. Larger members would vote for higher levels of SITG while smaller members would vote for lower SITG levels. At a *heterogeneous CCP*, reaching a consensus on an optimal level of SITG can become complicated as members with different levels of DF would vote for different levels of SITG.[5]

[5] Note that control in the form of voting rights may be allocated according to a simple one-member-one-vote rule.

Concluding Remarks

- The proposed framework is grounded in ICCs that address risk management agency problems. SITG formulations are simple and readily implementable using data available to regulators. Comparing our SITG formulations with empirical evidence, we conclude that SITG should be regulated, and that CCPs may need to allocate more capital to default waterfalls.
- Resilience of CCPs that will be at the center of the UST market is of critical importance. The first recommendation of G30 in 2021 was that the Federal Reserve should create a Standing Repo Facility (SRF) that provides very broad access to repo financing for UST securities.
- The SRF that was created in 2021 did not provide the very broad access due to concerns about creating moral hazard problems that would increase systemic risks. The G30 have suggested that this moral hazard could be mitigated by centrally clearing repos provided by the SRF. Our investigation indicates that this agency problem may be counteracted when CCP risk management agency problems are mitigated effectively.

Appendix

Cover-n Case

- Total DF is represented by $D_{s,n} = E_1 + E_2 + ... + E_n$. $L^1_{i,n}$ represents member i's loss conditional on member 1's default. S_n ($\tilde{S}_n$) represents the first (second) layer of SITG.
- We formulate S_n that correspond to the target loss probability $\pi_n = P(L^1_{i,n} > 0)$. The first layer of SITG is formulated according to

$$S_n = \left[\left(\frac{(\frac{q}{\pi_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) - c_1 \right] D_{s,n},$$

  and the second layer is formulated as

$$\tilde{S}_n = \left[\left(\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - (\frac{q}{\pi_n})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) + c_1 - 1 \right] D_{s,n}.$$

- $\tilde{\pi}_n$ is the target loss probability associated with $\tilde{S}_n$, $\tilde{\pi}_n = P(\tilde{L}^1_{i,n} > D_{i,n})$. $\tilde{L}^j_{i,n}$ is the total loss to i's prefunded and unfunded DF conditional on the default of j. Member i's DF is denoted by $D_{i,n}$.

Numerical Example

$\frac{S+\tilde{S}}{D}$	α	q (bps)	q_D (bps)	$\tilde{\pi}$ (bps)
0.67	2	100	50	35
0.18	2	100	50	45
0.09	2	200	100	95
1.62	2	100	30	10
2.51	2	100	80	50
17.32	2	100	80	10
3.44	3	100	50	10
0.61	3	100	50	35
1.34	3	100	30	10
0.36	4	100	50	40
0.59	4	100	50	35
0.67	4	80	60	50
3.17	4	70	40	10
0.3	4	100	80	75
0.16	5	100	50	45
0.57	5	100	50	35
2.93	6	100	50	10
0.57	6	100	50	35
0.62	6	100	80	70

Figure: Total SITG, $S + \tilde{S}$, as a fraction of default fund D for different values of parameters α, q, q_D, and $\tilde{\pi}$.

CCP Objective Function

Suppose that $E_t = S + \tilde{S} + E_s$ is the total capital of the investor-owned CCP. Conditional on j's default, consider CCP's loss in excess of members' resources and E_t,

$$L_e = (U_j - M_j - D - \beta(D - D_j) - E_t)^+.$$

The private profit-seeking objective of the CCP would be to maximize

$$\phi V - E[L] - E[L_e] - c(E_t) - c_p Q(S, \tilde{S}, E_s),$$

$c(E_t)$ is the social cost of capital, c_p is the private cost of a CCP failure, and $Q(S, \tilde{S}, E_s)$ is the probability of such a failure.[6]

Suppose that E_t is set by regulators but the allocation of it to S, $\tilde{S}$, and E_s is left to the CCP. Then, the CCP maximizes its objective by setting $S = \tilde{S} = 0$.

[6] Greenwood et al. (2017) use similar assumptions to formulate the cost of capital and bank failure in their setting.

Optimal Capital Regulation

- Social welfare can be represented by

$$\phi V - E[L] - E[L_e] - c(E_t) - c_s Q(S(\pi), \tilde{S}(\tilde{\pi}), E_s), \tag{16}$$

  where $S(\pi)$ can be viewed as a decreasing function of π, and given π, $\tilde{S}(\tilde{\pi})$ is a decreasing function of $\tilde{\pi}$. Given members' IM and DF assets, the social planner's objective would then be to find the optimum SITG and E_s that maximize (16). If the Pareto tail exponent, α, is also given, the social planner's problem can be equivalently viewed as finding the optimum π, $\tilde{\pi}$, and E_s that maximize (16). Optimal SITG can then correspond to target loss probabilities under which some of the ICCs may be satisfied and some may not.

- SITG can be incorporated into policymakers' *improved objective function*. S and $\tilde{S}$ are linked to a set of incentive compatibility constraints under which some of the CCP risk management agency problems can be mitigated.

Client Clearing Implications

- That the CCP has small default loss exposure $P(L_0^j > 0) \leq q_D < q$, relies on the assumption that DF has been sized adequately. These loss probabilities need not remain small if DF does not capture client clearing risks properly. For instance, if in estimating DF, the CCP's exposure to member 1, U_1, does not take into account portfolios that member 1 has cleared for customers, $P(L_0^1 > 0)$ can exceed q_D and q.

- Suppose that member 1 defaults and its IM covers losses associated with member 1's *house account*. Over a period of time till *client accounts* can be ported to a non-defaulting member, the CCP may need to make payments to member 1's customers. If DF is not sized properly to cover losses that could arise due to member 1's default, the resilience of the CCP can be adversely impacted.

- **The default waterfall should evolve proportionately to the risk profile of the CCP. Increased client clearing should increase IM, DF, and SITG adequately.**