Momentum Through the Looking Glass

Contents

Introduction 1
1 Data 1
2 Momentum using Options 2
2.1 Basic Analysis 2
2.2 Implications for Momentum 5
3 Momentum in a Portfolio 6
3.1 Risk Measures 6
3.2 Implications in Mean-Var Framework 7
3.3 Practical Constraints 7
4 Enhancements to TSMOM investing 7
4.1 Vol and VaR targeting 7
4.2 Adjusting Mean-Var Objective Function 9
4.3 Regime-switching VaR 9
5 Conclusion 10
References 10

Introduction

Momentum strategies have steadily gained popularity in recent years. There are actually two distinct momentum strategies: cross-sectional momentum and time-series momentum. The Carhart momentum factor commonly referenced in Fama-French literature is cross-sectional momentum. Stocks are ranked cross-sectionally at a point in time based on their last 12 months of returns [1]. For time-series momentum, the recent moving average of an asset's return is compared with a longer history of its moving average. This paper will focus on the time-series momentum strategy.

The paper by Moskowitz, Ooi and Pedderson [2], talks about TSMOM as a strategy that is pervasive across many asset classes. They claim that the strategy provides diversification in bad times. This is confirmed by several other authors who show both theoretically and empirically that the return profile of momentum is convex, similar to a straddle option. Because the strategy generates positive returns in good times and hedges in bad times, momentum is considered a market anomaly due to behavioral reasons rather than a risk premium [3].

In this paper, we investigate some ways to replicate time series momentum (TSMOM) using at the money call/put options and straddles and discover insights that will help risk manage these strategies. We investigate the distribution of TSMOM returns and note the implications it has from a mean variance utility perspective and also for traditional risk measures like VaR and CVaR. We also contrast its behavior in different volatility regimes.

The rest of the paper is organized as follows: Section 1 will talk about the data and sources. Section 2 will talk about the construction of four portfolios prescribed in the problem statement. We report their characteristics and provide some analysis around them. We will also explore what are the implications for momentum. Section 3 provides an analysis of the risk embedded in momentum portfolios and talk about how this fits into a traditional Value at Risk (VaR) and Mean Variance (MV) framework. Finally, Section 4 describes the enhancements that we provide to the momentum strategy itself and to the calculation of VaR.

1. Data

We obtained the following data from Bloomberg: implied volatility for at-the-money calls and puts, S&P 500 total index (including dividends), S&P 500 price index (ex-dividends). We use approximately 12 years of data, beginning June 2005 and ending August 2017. In order to build the volatility surface, we choose the 30-day and 90-day implied volatility for 90%, at-the-money and 110% moneyness. From the implied volatility, we used linear interpolation to construct a volatility surface along the dimensions of moneyness and time to maturity.

The S&P 500 total index is used to calculate both the moving-average-cross-over signals as well as the returns of the momentum portfolio since we are directly investing in the underlying and will receive dividends. The S&P 500 price index is used to price options and calculate the returns of the option strategies, since we do not receive dividends for options.

2. Momentum using Options

This section describes the four portfolios that the problem statement asks us to build. The portfolios are based on 60-day moving average (MA_{60}) and 120-day (MA_{120}) moving average of the S&P500 index. If $MA_{60} > MA_{120}$, the current trend is positive, and we would want long exposure to the underlying. Similar logic follows if the trend is negative. Throughout this paper, we make the following assumptions: no transaction cost or financing cost, fractional holdings allowed and continuous range of option strike and maturity.

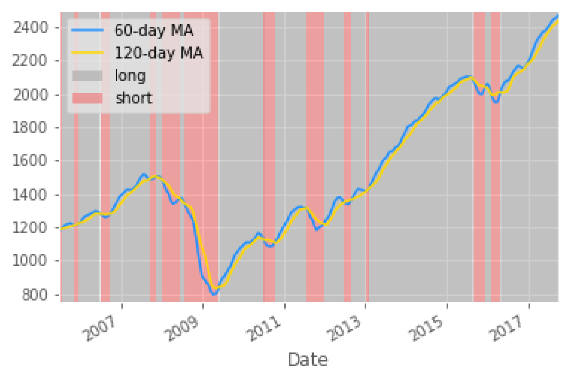

Figure 1. 60 and 120-day moving averages

- The momentum portfolio consists of being long one unit of the index when $MA_{60} > MA_{120}$ and being short one unit when $MA_{60} < MA_{120}$. Henceforth, we will call this TSMOM Strategy.

- The first options strategy uses at the money (ATM) call and put options. If $MA_{60} > MA_{120}$, we buy one ATM 90-day call and if $MA_{60} < MA_{120}$, we buy one ATM 90-day put; we rebalance daily. Henceforth, we will call this the LCP (Long Call Put) Strategy.

- We interpret the second options strategy as buying a 90-day ATM straddle at the beginning, rebalanced daily. Henceforth, we will call this the LS (Long Straddle) Strategy.

- For the third options strategy, we buy one 90-day ATM straddle at the beginning, valued using the implied volatility interpolated from volatility surface. We then delta hedge to 0 and rebalance the hedge daily. We roll the straddle over once the straddle matures. Henceforth, we will call this SRB (Straddle Rebalance) Strategy.

- An additional portfolio that we build is very similar to LCP, but instead of going long one call we go short a put and instead of long one put, we go short one call. Henceforth, we will call this SCP (Short Call Put) Strategy.

This is summarize in Table 1. The next subsection provides some summary statistics on the index, the TSMOM portfolio, and the four portfolios that we construct.

Acronym	Strategy
TSMOM	Time Series Momentum
LCP	Long Call Put
LS	Long Straddle
SRB	Straddle Rebalance
SCP	Short Call Put

Table 1. Strategies

2.1 Basic Analysis

We tabulate some summary statistics for the raw returns of the four portfolios that we constructed in Table 2. Particularly, we look at the first four moments of the return distribution and the Sharpe ratio.

The index, TSMOM, LCP and SCP have positive returns while the straddle strategies have negative returns. This makes sense because straddle strategies are short implied volatility and long realized volatility. This generates consistent negative returns due to the volatility risk premium [4].

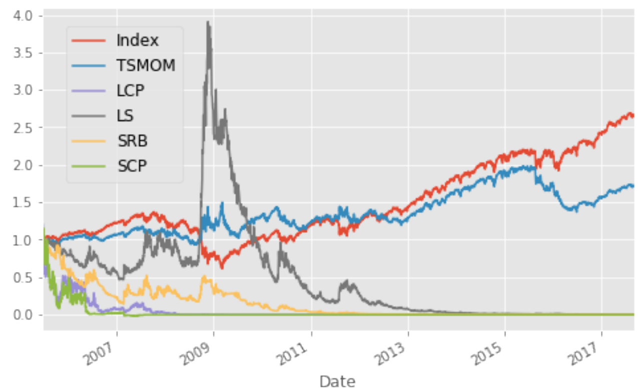

Figure 2. Five portfolio cumulative return

	Index	TSMOM	LCP	LS	SRB	SCP
Ann Ret (%)	9.93	6.35	67.23	-28.51	-20.73	110.22
Vol (%)	19.35	19.35	209.17	74.48	100.21	253.56
SR	0.51	0.33	0.32	-0.38	-0.21	0.43
Skew	-0.10	-0.15	0.75	1.95	4.14	-1.00
Kurtosis	11.81	11.80	5.13	9.00	46.74	4.15
Max	0.12	0.09	1.15	0.40	1.10	0.50
Min	-0.09	-0.12	-0.62	-0.15	-0.21	-1.45

Table 2. Summary statistics

	Index	MOM	LCP	SCP	LS	SRB
Index	1	-0.28	-0.21	0.22	-0.70	-0.60
MOM		1	0.92	0.82	-0.11	-0.16
LCP			1	0.82	0.00	-0.07
SCP				1	-0.55	-0.54
LS					1	0.82
SRB						1

Table 3. Correlation Matrix

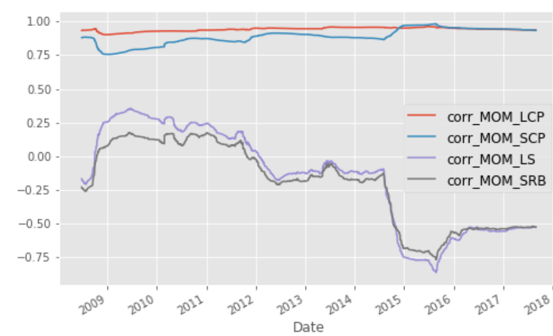

Figure 3. Three year rolling correlations

TSMOM has slightly negative skew as shown in Table 2.[1]

Of the portfolios prescribed in the problem statement, LCP has the best performance in terms of absolute return. However, this is a highly levered strategy since it uses options. On a risk-adjusted basis, assuming a risk-free rate of 0%, the TSMOM portfolio has the highest Sharpe ratio of 0.33. It is important to note that all of these strategies underperform the index in terms of risk adjusted returns, which has a Sharpe ratio of 0.51.

As seen in Table 2 and Figure 3 it is clear that the LCP and SCP have very high correlation with TSMOM. Also, when we condition on being long or short, we see that the correlations of LCP and SCP are positive and negative respectively. This strenghtens our claim that LCP and SCP are pretty close to momenntum. It is expected due to the payoff structure: when the momentum signal works, LCP has the same payoff as TSMOM and SCP profit is capped by option premium; when it does not work, SCP has the same payoff as TSMOM and the loss of LCP is capped by option premium. Hence we regard them as a levered version of momentum.

Observing the cumulative returns series in Figure 2, it is quite interesting that all the options strategies crash. For the straddle strategies, this makes perfect sense due to the highly negative returns coupled with high leverage. For the SCP and LCP strategies, we make an argument based on their similarity to momentum. Jusselin et al. (2017) [3] find that momentum is highly sensitive to leverage. Under the theoretical model that the asset price follows geometric Brownian motion with trend, they show that additional leverage beyond a certain optimal leverage ratio starts decreasing the P&L. Moreover, the probability of ruin increases rapidly from near 0% to near 100%. This is directly observed in our empirical results.

Expanding on the idea of LCP and SCP being a levered version of momentum, if we control for option beta, the strategies look very much like momentum. The beta of a call option was computed in

[1]This is consistent with the data that AQR capital has released for the original Time Series Momentum paper on its website. The data can be accessed here: `https://www.aqr.com/library/data-sets/time-series-momentum-original-paper-data` The skew of the TSMOM^{eq} series that they construct is -0.15.

Black and Scholes (1973)

[5] as:

$$\beta_{\text{call}} = \frac{S}{P_{\text{call}}} \Delta_{\text{call}} \cdot \beta_S \tag{1}$$

where P_{call} is the price of the call, S is the price of the underlying (S&P 500), Δ_{call} is the delta of the call and β_S is the beta of the underlying, which is equal to 1 for the S&P 500 by definition. The intuition behind this measure is that it is exposure per cost. We calculate the cumulative return of the LCP and SCP index after scaling the daily return by the absolute values of the beta.

The reason to do so is that it brings the Δ exposure to ± 1, so that its return is comparable to the momentum portfolio. The difference then comes from exposure of the options to the other Greeks that are not present in the momentum portfolio. One way to neutralize these exposures is to sell 1/2 LS for every LCP unit, which effectively reduces to holding $\pm 1/2$ a call option and $\mp 1/2$ put, which is then equivalent to holding $\pm 1/2$ unit of the index, or holding 1/2 TSMOM. We obtain that the β of the call and put options is roughly 20. The scaled series have been plotted in Figure 4

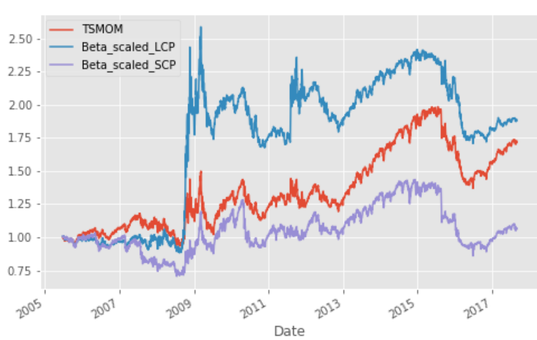

Figure 4. Beta scaled option strategies

As seen clearly in Figure 4, LCP and SCP are very similar to TSMOM after they have been scaled by option β, and the difference comes from the non-zero Greeks.

Looking next at the distribution of the returns, we see that none of these portfolios are close to being normal. Plotting the QQ-plot for all of them, this becomes very clear. TSMOM and the index

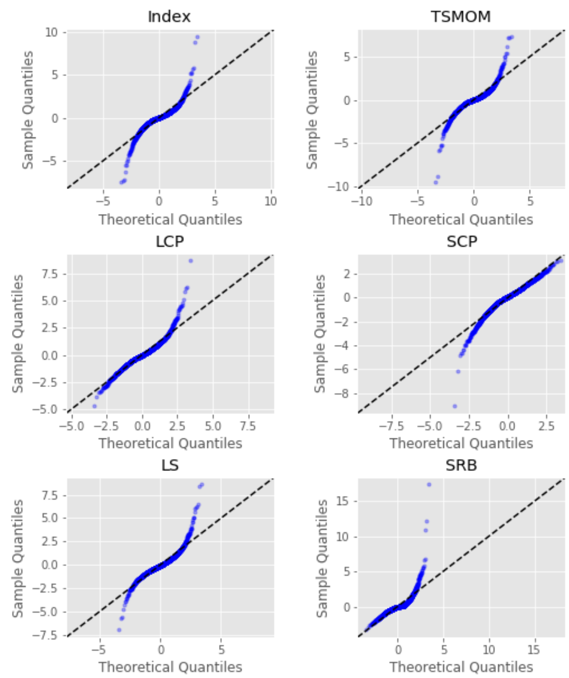

Figure 5. QQ-Plots

have very high excess kurtosis while LCP and SCP have high skew – positive and negative respectively.

The intuition behind skew of LCP and SCP is that LCP is levered on the upside (due to the non-linear nature of the call/put options), as seen in Figure 6 [6]. When the momentum indicator works, the option provides leverage and when it does not, the losses are capped by the option premium – hence the positive skew.

Another notable observation is the low correlation of LS and SRB with TSMOM. The major reason is that the long straddle bets on volatility without regard to price movement direction. In contrast, TSMOM bets on direction based on the assumption that future price movement direction is the same as the observed trend. And that explains the low correlation. The major difference between LS and SRB is delta hedging. When the stock price moves up, the option delta increases which leads to a sell-off of stocks to neutralize delta and vice versa. In other words, the delta hedging position is akin to buying low and selling high, which is the opposite of momentum. This is why the SRB correlation with TSMOM is slightly lower than the LS correlation.

The literature on trend following strategies claims that long term momentum/trend following strategies have a convex payoff over a long term. Since our strategies rebalance frequently, our results diverge from this observation. Also, Jusselin et. al have shown that shorter term signals do not show as much convexity as longer-term moving signals [3]. We are using 2-month and 4-month moving averages, which is a shorter-term signal. Another possible reason is that there are idiosyncrasies in the sample period of our data. We use only the past 12 years of data, while many of the empirical studies look at a much longer term.

If we were to pick one strategy that best represents TSMOM return distribution out of all these strategies, we would pick a delevered SCP because it has very high correlation to TSMOM, has negative skew and some excess kurtosis.

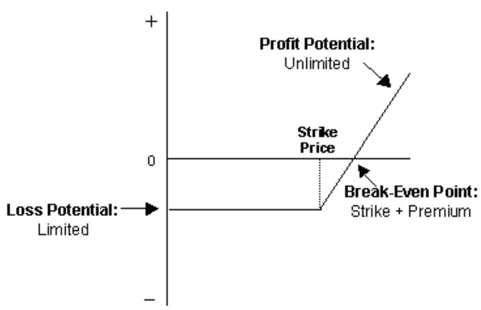

Figure 6. Non-linear payoff (from CBOE 2018)

2.2 Implications for Momentum

In this section, we explore the implications of our findings for momentum strategies. For analyzing TSMOM, we start by fitting a regime-switching model that was introduced in by Hamilton in [7]. We believe that the returns will have distinctive behavior under different regimes and propose a simple regime-switching model

$$r_t = \mu_{s_t} + \sigma_{s_t} \varepsilon_t, \qquad (2)$$

where r_t is the daily return of S&P 500 total index, $s_t = 0, 1$ is the regime indicator, which is a time-homogeneous Markov chain, ε_t follows standard normal distribution, and μ_{s_t} and σ_{s_t} is the mean and volatility at state s_t respectively.

	Value (Ann.)	t_{stat}
μ_1	13.4%	5.721
μ_2	-15.1%	-1.266
σ_1	10.8%	18.844
σ_2	31.7%	14.791

Table 4. Parameter of the regimes

	High Vol	Low Vol
Cum Ret	-45%	211%
SR	-0.44	1.13
Volatility (Ann.)	35%	11%

Table 5. TSMOM returns in Vol Regimes

From the empirical study by Hardy [8], the model performance of a two-state regime-switching model for equity indices is good and increasing the number of states does not improve the performance significantly. Therefore, we adopt a two-state regime model. Observing the volatilities, we can say that one is low-volatility state and the other is high-volatility state.

We assume that the process is in the high-volatility state when the smoothed probability of being in this state is greater than 80%. Table 5 shows that TSMOM does very well in the low-volatility regimes and under-performs in high-volatility regime. This is consistent with what has been reported by Pettersson (2014) [9].

Another piece of analysis we did is to regress the returns of TSMOM against the returns of VIX in the different regimes. We see that the loading of TSMOM on the implied volatility (VIX) changes between the regimes, being short VIX in the low volatility environment and long in the high volatility one. It also verifies the low correlation of LS and SRB as discussed in section 2.1, as LS and SRB are always long volatility. The regression coefficients and t-statistics are reported in Table 6.

To illustrate these results, we plot the rolling 1-year correlation of VIX and TSMOM along with VIX and shade high volatility regimes in Figure 7.

The fact that momentum does poorly when volatil-

Figure 7. Correlation of VIX and TSMOM in volatility regimes

	Low Vol		High Vol	
	Coeff.	t_{stat}	Coeff.	t_{stat}
const	0.00	4.69	-0.001	-1.276
VIX	-0.06	-32.12	0.054	6.642

Table 6. Regression of TSMOM on VIX

ity increases can be used to our advantage. We describe in Section 3, how we can improve the performance of the momentum strategy by scaling using volatility.

3. Momentum in a Portfolio

3.1 Risk Measures

	Index	TSMOM	LCP	LS	SRB	SCP
MaxDD (%)	55.25	31.86	100.00	99.97	99.97	101.68
MaxDD Dur (Years)	1.42	0.94	11.36	8.70	12.10	1.94
90VaR (%)	-1.18	-1.22	-14.72	-4.65	-5.63	-19.26
95VaR (%)	-1.81	-1.81	-20.35	-5.98	-7.61	-27.86
99VaR (%)	-3.51	-3.50	-31.45	-8.43	-12.30	-47.70
90CVaR (%)	-2.23	-2.20	-22.36	-6.51	-8.62	-31.98
95CVaR (%)	-3.01	-2.91	-27.45	-7.73	-10.66	-41.01
99CVaR (%)	-5.25	-5.02	-38.82	-10.56	-14.37	-62.33

Table 7. Risk measure values

The VaR and CVaR calculations in Table 7 are all historical measures. In the enhancements section, we will present VaR and CVaR based on simulations with the fitted regime-switching model. From a risk perspective, it is clear that none of these options strategies are a good representation of future risk for momentum. All strategies are highly levered and have a maximum drawdown of close to 100%. Similarly, the VaR and CVaR are multiples higher for all the options strategies. The duration of the maximum drawdown is also an order of magnitude higher for all of the portfolios with the exception of the SCP strategy.

	Delevered LCP	Delevered SCP
MaxDD (%)	35.23	40.06
MaxDD Dur (Years)	1.71	1.56
90VaR (%)	-1.07	-1.39
95VaR (%)	-1.71	-2.19
99VaR (%)	-3.77	-3.97
90CVaR (%)	-2.14	-2.54
95CVaR (%)	-2.94	-3.38
99CVaR (%)	-5.38	-5.43

Table 8. Risk measures for delevered strategies

To get comparable risk measures, we apply the aforementioned β delevering strategy for the LCP and SCP portfolios. Upon delevering, we can see that both strategies give a very good representation of future risk for momentum under the measures of max drawdown, duration, VaR and CVaR.

These are simply the statistical measures of risk, but we also want to pay attention to the risk factors for momentum. The obvious risk is trend reversal, but because time-series momentum is a market anomaly stemming from behavioral reasons, there is always the risk that trend disappears as the market gradually becomes more efficient. Another consideration is that although TSMOM can hedge against bad times, it is only able to capture more gradual declines in the market. This is due to the filtering problem, as the 60-day moving average is not able to capture a sharp decline over a few days fast enough to trade on the signal.

3.2 Implications in Mean-Var Framework

In the classic paper [10], Markowitz (1952) lays the foundation for what is now known as the mean-variance (MV) framework. Investors are risk averse, which means that for investing in any risky portfolio, they want to get compensated in returns for the risk that they're taking [11]. In this framework, investors care only about the mean and the variance. The optimal portfolio in a MV framework is the one that has the highest Sharpe ratio. Hence, we would choose TSMOM, which has the highest Sharpe ratio.

In a later paper, Malamud (2014) [12], describes what happens when we include non-linear type of payments (like options) to the MV framework. He claims that is a non-trivial problem: the non-linearity of the payoffs introduces the problem of estimating higher moments of the distribution - skewness and kurtosis. As described in previous sections, TSMOM has negative skewness and positive kurtosis. In section 4 we will introduce two ways of expanding the framework: Greeks efficient portfolios and Mean-CVaR analysis.

3.3 Practical Constraints

In terms of implementing these strategies, let's start with TSMOM. This strategy has a relatively low turnover - 20 total trades in the time horizon studied, which translates into low transaction costs. Because we are working with the S&P 500 index, we have low transaction costs and a very liquid market which can be implemented by simply buying or selling an ETF.

For the LCP and SCP strategies, both are naked strategies (no delta hedging with the index). Also, for LCP we need to find sellers of options with strikes that have extra granularity: we are buying new ATM calls or puts everyday, each with a new 90-day maturity. This is not possible in normal exchanges (CBOE or CME), thus we require an OTC dealer, which it will be more expensive. It could also be hard to realize the daily P&L since finding a buyer for a 89-day option could be difficult. This also applies to the straddle strategies. For the last strategy, we have the impractical consequences of daily hedging: transactions cost and shorting costs.

On the other hand, the original time series momentum factor by Moskowitz et.al (2012) [2] is constructed differently: normally we will be looking for the 12-month moving average (ignoring the most recent month) to capture the long term movement and the 1-month moving average to capture the short mean reversal . This way of building the series have been proven to be an investing factor that can be exploited in equities, indices, commodities, and other asset classes.

4. Enhancements to TSMOM investing

Although the momentum generates stunning performance, the large maximum drawdown, negative skew and excess kurtosis urges us to take care of the risk associated to the momentum strategy. This section explores several approaches to improve risk management.

4.1 Vol and VaR targeting

The TSMOM that we implement has excess kurtosis and doesn't beat the index in terms of Sharpe ratio. As seen in Section 2, TSMOM tends to do poorly in times of high volatility. We take advantage of this information by scaling down the allocation when the market seems to be in the high-volatility state.

First, we try to replicate what Pedro Barroso et al. did in their paper [13]. The core idea of managing momentum risk that they propose is that the realized variance of daily returns is highly forecastable. In order to control the risk exposure in the momentum strategy, we scale our holdings in the index to match a target volatility which we keep constant over time: σ_{target}.

The monthly forecast $\hat{\sigma}_t^2$ is computed from daily returns in the last six months. Let $\{r_t\}_{t=1}^{T}$ be the monthly returns of momentum strategy and $\{r_d\}_{d=1}^{D}$, $\{d_t\}_{t=1}^{T}$ be the daily returns and the time series of the trading days. Then, the variance forecast is

$$\hat{\sigma}^2 = \frac{21}{126} \sum_{j=0}^{125} r_{d_{t-1-j}}^2 \tag{3}$$

Since we want to manage the risk of momentum strategy and not just scale to amplify our positions in times of low volatility , the factor will be capped at 1 to limit the leverage. The scaled portfolio holding will be:

$$h_t = \min\left(\frac{\sigma_{\text{target}}}{\hat{\sigma}_t}, 1\right) \tag{4}$$

The optimal portfolio using MV framework, is [14]:

$$h^* = \frac{\mu - r_f}{2\lambda\sigma^2} \tag{5}$$

If we set $\frac{\mu - r_f}{2\lambda} = \sigma^2_{\text{target}}$, the result reduces to exactly the volatility targeting we have done; without the cap at 1. Another insight we get from this is that we should increase/decrease the target volatility based on the expected return. We should take more risk when the expected return is higher.

We set a target volatility of 10%, which is around half of the TSMOM volatility and present the results below.

	Vol scaled TSMOM	TSMOM
Cum Ret	76.52%	72.42%
Ann Ret	5.69%	6.35%
Volatility (ann.)	10.33%	19.35%
SR	0.52	0.33
MaxDD	-21.20%	-31.86%
MaxDD Dur (year)	0.94	0.94
Skew	-0.15	-0.15
Kurtosis	3.33	11.79
Max	4.32%	9.03%
Min	-3.64%	-11.58%
99VaR	-1.87%	-3.58%
95VaR	-1.07%	-1.81%
90VaR	-0.74%	-1.25%

Table 9. Targeting Volatility

The result in table [9] shows that the Sharpe ratio improves from 0.33 for simple momentum to to 0.52 for its volatility scaled version. The excess kurtosis drops from 11.79 to 3.33 and the maximum drawdown of -31.86% improves to -21.20%.

However, we did not get the same improvement in skewnewss as Barroso et al. [13], as the left skewness did not decrease. This is mainly because

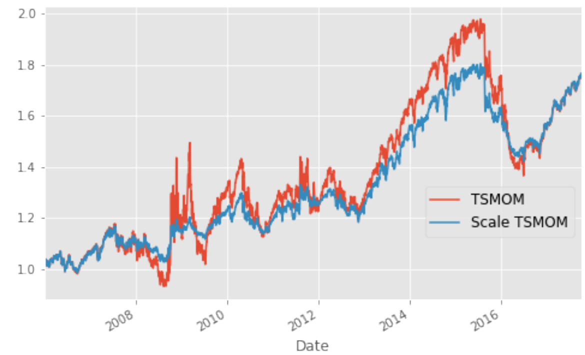

Figure 8. Vol scaled TSMOM vs TSMOM

we apply the strategy to a different momentum factor. In the paper, they use the traditional winner-minus-loser portfolio (cross-sectional momentum), while we are using the time-series momentum factor explained in [2]. The skewness of our original portfolio is small enough, -0.15, compare to the plain winner-minus-loser's -2.47, number reported by Barroso et al. [13].

Scaling by VaR would output the same results as scaling by volatility if the returns are normally distributed. This can be seen in the following equation:

$$\text{VaR} = \mu + \sigma\Phi^{-1}(\alpha) \tag{6}$$

where $r_t \sim N(\mu, \sigma^2)$ and $\Phi(\cdot)$ is the CDF of a standard normal.

But, we showed that TSMOM returns are not normally distributed. They have excess kurtosis and also have negative skew. So, scaling by historical VaR could be a useful exercise. This is the second method we try to risk manage momentum strategy. We calculate the daily 5% historical VaR uisng the data from the last six months. Then we scale the portfolio weights to 1% daily VaR and cap the scaling at 1. So, the amount we hold in the index (long/short) is:

$$h_{\text{scaled}} = \frac{\text{VaR}_{\text{target}}}{\text{VaR}_{t,5\%}} \tag{7}$$

The results for the VaR targeted momentum (targeted to 1% daily VaR) has been reported in Table 10.

Both scaling methods provide us with improved performance. However, in terms of picking one

	VaR scaled TSMOM	TSMOM
Cum Ret	63.39%	72.42%
Ann Ret	4.55%	6.35%
Volatility(ann.)	10.13%	19.35%
SR	0.45	0.33
MaxDD	-20.15%	-31.86%
MaxDD Dur(years)	0.94	0.94
Skew	-0.26	-0.15
Kurtosis	4.56	11.79
Max	3.91%	9.03%
Min	-4.96%	11.58%
99VaR	-1.79%	-3.50%
95VaR	-1.03%	-1.81%
90VaR	-0.74%	-1.22%

Table 10. Targeting VaR

between the two methods, volatility-scaling will fit better in our framework since volatility scaled TSMOM does not increase the skewness and has higher Sharpe ratio.

4.2 Adjusting Mean-Var Objective Function

Mean-variance optimization is a good approximation in most cases, but Cremers, Kritzman and Paige (2003) [15] suggest that if investors have quadratic utility, they're indifferent to higher moments. However, higher moments become important when there is a risk of non-survival. This is exactly the case with TSMOM. The risk of TSMOM is due to exposure to higher moments, and also we have seen that it has large drawdowns. Thus, the objective function must be adjusted for it.

Our first approach will be to build the framework of Greek efficiency, similarly to the work of Malamud (2014) [12]. Using a Taylor expansion, we can decide over the number of sensitivities that the investors want to manage and budget. Assuming investors have a CRRA utility function with risk aversion γ and we can estimate all co-movements (covariances, co-skewness or co-kurtosis), we can generate an optimal Greek, which would be similar to the portfolio weights of Markowitz (1952):

$$\Gamma = \gamma^{-1}\Sigma_t^{\mathrm{T}}\mu \tag{8}$$

where Σ_t is a linear matrix that calculates co-movements in terms of higher moments. Malamud (2014) [12] found that this way of budgeting risk present higher Sharpe ratio than a classical MV framework.

The second approach for handling the risks associated with doing a naive MV framework is doing a Mean-CVaR [12]. Because options returns present non-linear payoff, and our TSMOM also have non-normal returns, we need to use asymmetric risk measures. Between VaR and CVaR as tail risk measure, CVaR is a coherent risk measure [12]. Hence, according to Xiong and Idzorekin (2010) [16], this method will prefer to choose assets with positive skewness, small kurtosis, and low variance. From this we can create a new performance ratio: the STARR ratio (Stable Tail Adjusted Return Ratio), first introduce by Martin, Rachev and Siboulet (2003) [17]:

$$\mathrm{STARR}(w) = \frac{r_p(w) - r_f}{\mathrm{CVaR}_\alpha} \tag{9}$$

The results for the STARR for our strategies are reported in Table 11, assuming 0.0% as the risk-free rate:

	TSMOM	LCP	LS	SRB	SCP
90-STARR	2.97	3.01	-4.38	-2.40	3.45
95-STARR	2.24	2.45	-3.69	-1.94	2.69
99-STARR	1.30	1.73	-2.70	-1.44	1.77

Table 11. STARR values for different levels of confidence

Using this framework, the best strategy among the 4 presented in the problem statement is LCP, and we can observe how non-normal the distributions are and how they differ in the tails. TSMOM is penalized for having negative skewness, while the LCP strategy is rewarded for having positive skewness.

4.3 Regime-switching VaR

Due to the distribution of TSMOM, we expect that historical VaR understates the risk exposure to extreme event. As discussed in section 2.2, the cluster of low-volatility and high-volatility returns inspires us to re-evaluate VaR under regime-switching model.

To evaluate the VaR measure, we adopt the classical rolling forecast, which will estimate the model parameters in a specified lookback period and evaluate VaR measure for the future periods and iterate the procedure by rolling the lookback period one year forward till the end of the historical data. Exception number test is used to evaluate VaR performance. The test reports the number of realized return bigger than VaR and it is one of the most popular because the Basel Committee[18] has adopted it to categorize the internal models into different zones (green, yellow, red). As the regime-switching model requires long time series to recognize different regimes, we used 8-year historical returns in the lookback period to estimate model parameters, and simulate one-year daily return of the index and calculate momentum returns based on Portfolio 1 construction. And 95% VaR is calculated based on terminal simulation returns and we call it RS VaR (Regime-switching VaR) and the exception test is performed in the following year. For comparison, historical VaR is also computed in the estimation period.

Year	Hist VaR	No. Exceed	% Exceed	RS VaR	No. Exceed	% Exceed
2008	-1.66%	46	18.2%	-1.81%	39	15.4%
2009	-1.84%	29	11.5%	-2.18%	24	9.5%
2010	-2.02%	13	5.2%	-2.23%	9	3.6%
2011	-1.85%	22	8.7%	-2.01%	17	6.7%
2012	-1.98%	4	1.6%	-2.27%	3	1.2%
2013	-2.05%	2	0.8%	-2.24%	2	0.8%
2014	-2.08%	3	1.2%	-2.15%	1	0.4%
2015	-2.09%	5	2.0%	-2.12%	4	1.6%
2016	-2.08%	6	2.4%	-1.98%	6	2.4%
2017	-1.82%	0	0.0%	-1.76%	1	0.4%

Table 12. 95% VaR measures in rolling forecast

For 95% VaR with 1-year realized returns (252 observations), we expect 12 exceptions but Table 12 shows that the exceptions are out of expectation from 2008 to 2011, which is regarded as a stressful period. Under such stressful period, regime-switching VaR is prone to to correct the underestimation issues by reporting less exception number than that of historical VaR. And the results are also consistent with what R. Kawata and M. Kijima [19] found.

5. Conclusion

Many hedge funds and asset managers allocate heavily to momentum strategies. These strategies have been pretty successful. However, there is a need for better risk management of these strategies, given that they suffer from large drawdowns, perform poorly when high volatility prevails in the market and exhibit large kurtosis with some negative skew.

We show that a simple TSMOM strategy can be replicated using call and put options; but we need to control for the leverage of the strategy. We addressed the issues that momentum strategies suffer from while trying to preserve the profitability of the strategy. We use the high persistence of volatility to scale back our bets on momentum in high volatility times, propose a more efficient way to calculate VaR that takes into account the fact that there are two volatility regimes and the behavior of momentum switches in these regimes. We acknowledge that although mean-variance framework is sufficient in most cases, if we consider momentum as an asset, we need better frameworks that quantify this risk because of TSMOM's drawdowns. We propose to use a CVaR optimized framework to decide our optimal portfolio.

References

[1] Narasimhan Jegadeesh and Sheridan Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, 48(1):65–91, 1993.

[2] Tobias J Moskowitz, Yao Hua Ooi, and Lasse Heje Pedersen. Time series momentum. *Journal of financial economics*, 104(2):228–250, 2012.

[3] Paul Jusselin, Edmond Lezmi, Hassan Malongo, Côme Masselin, Thierry Roncalli, and Tung-Lam Dao. Understanding the momentum risk premium: An in-depth journey through trend-following strategies. 2017.

[4] Bjørn Eraker. The volatility premium. *Manuscript*, 2008.

[5] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654, 1973.

[6] CBOE. Buying index calls, 2018.

[7] James D Hamilton. Analysis of time series subject to changes in regime. *Journal of econometrics*, 45(1-2):39–70, 1990.

[8] Mary R Hardy. A regime-switching model of long-term stock returns. *North American Actuarial Journal*, 5(2):41–53, 2001.

[9] John Pettersson. Time series momentum and volatility states. 2014.

[10] Harry Markowitz. Portfolio selection. *The journal of finance*, 7(1):77–91, 1952.

[11] William F Sharpe. Mutual fund performance. *The Journal of business*, 39(1):119–138, 1966.

[12] Semyon Malamud. Portfolio selection with options. 2014.

[13] Pedro Barroso and Pedro Santa-Clara. Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120, 2015.

[14] Richard C Grinold and Ronald N Kahn. Active portfolio management. 2000.

[15] Jan-Hein Cremers, Mark Kritzman, and Sebastien Page. Portfolio formation with higher moments and plausible utility. *Revere Street Working Paper Series 272-12*, 2003.

[16] James Xiong and Thomas Idzorek. Mean-variance versus mean-conditional value-at-risk optimization: The impact of incorporating fat tails and skewness into the asset allocation decision. *Ibbotson Associates*, 2010.

[17] R Douglas Martin, Svetlozar Zari Rachev, and Frederic Siboulet. Phi-alpha optimal portfolios and extreme risk management. *The Best of Wilmott 1: Incorporating the Quantitative Finance Review*, 1:223, 2003.

[18] Basel Committee et al. Supervisory framework for the use of backtesting in conjunction with the internal models approach to market risk capital requirements. *Basel Committee on Banking and Supervision, Switzerland*, 1996.

[19] Ryohei Kawata and Masaaki Kijima. Value-at-risk in a market subject to regime switching. *Quantitative Finance*, 7(6):609–619, 2007.